



Three-Dimensional Photogrammetry for Laboratory Applications



By

Nabih M. Alem

Aircrew Protection Division

19950306 018

December 1994

Approved for public release; distribution unlimited.

United States Army Aeromedical Research Laboratory
Fort Rucker, Alabama 36362-0577

Notice

Qualified requesters

Qualified requesters may obtain copies from the Defense Technical Information Center (DTIC), Cameron Station, Alexandria, Virginia 22314. Orders will be expedited if placed through the librarian or other person designated to request documents from DTIC.

Change of address

Organizations receiving reports from the U.S. Army Aeromedical Research Laboratory on automatic mailing lists should confirm correct address when corresponding about laboratory reports.

Disposition

Destroy this document when it is no longer needed. Do not return it to the originator.

Disclaimer

The views, opinions, and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy, or decision, unless so designated by other official documentation. Citation of trade names in this report does not constitute an official Department of the Army endorsement or approval of the use of such commercial items.

Reviewed:

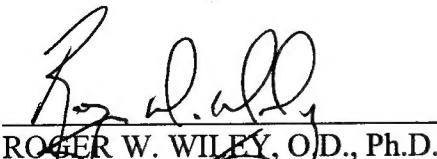


KEVIN T. MASON

LTC, MC, MFS

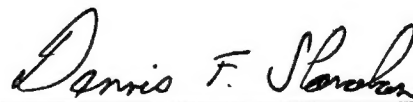
Director, Aircrew Protection
Division

Released for publication:



ROGER W. WILEY, O.D., Ph.D.

Chairman, Scientific
Review Committee



DENNIS F. SHANAHAN

Colonel, MC, MFS
Commanding

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1a. REPORT SECURITY CLASSIFICATION Unclassified			1b. RESTRICTIVE MARKINGS										
2a. SECURITY CLASSIFICATION AUTHORITY			3. DISTRIBUTION/AVAILABILITY OF REPORT Approved for public release, distribution unlimited										
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE			5. MONITORING ORGANIZATION REPORT NUMBER(S)										
4. PERFORMING ORGANIZATION REPORT NUMBER(S) USAARL Report No. 95-6			7a. NAME OF MONITORING ORGANIZATION U.S. Army Medical Research and Materiel Command										
6a. NAME OF PERFORMING ORGANIZATION U.S. Army Aeromedical Research Laboratory		6b. OFFICE SYMBOL (If applicable) ICMR-UAD-CI	7b. ADDRESS (City, State, and ZIP Code) Fort Detrick Frederick, MD 21702-5012										
6c. ADDRESS (City, State, and ZIP Code) P.O. Box 620577 Fort Rucker, AL 36362-0577		9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER											
8a. NAME OF FUNDING/SPONSORING ORGANIZATION		8b. OFFICE SYMBOL (If applicable)	10. SOURCE OF FUNDING NUMBERS										
8c. ADDRESS (City, State, and ZIP Code)		<table border="1"> <tr> <th>PROGRAM ELEMENT NO.</th> <th>PROJECT NO.</th> <th>TASK NO.</th> <th>WORK UNIT ACCESSION NO.</th> </tr> <tr> <td>62787A</td> <td>0162787A878</td> <td>EB</td> <td>142</td> </tr> </table>				PROGRAM ELEMENT NO.	PROJECT NO.	TASK NO.	WORK UNIT ACCESSION NO.	62787A	0162787A878	EB	142
PROGRAM ELEMENT NO.	PROJECT NO.	TASK NO.	WORK UNIT ACCESSION NO.										
62787A	0162787A878	EB	142										
11. TITLE (Include Security Classification) Three-dimensional photogrammetry for laboratory applications													
12. PERSONAL AUTHOR(S) Nabih M. Alem													
13a. TYPE OF REPORT Final		13b. TIME COVERED FROM _____ TO _____		14. DATE OF REPORT (Year, Month, Day) 1994 December									
15. PAGE COUNT 20													
16. SUPPLEMENTARY NOTATION													
17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)										
FIELD	GROUP	SUB-GROUP	measurements, imagery, computer program, calibration										
14	02												
20	06												
19. ABSTRACT (Continue on reverse if necessary and identify by block number)													
<p>The direct linear transformation (DLT) is a method that simplifies measurements of the three-dimensional coordinates of a point target in the laboratory using photographic two-dimensional imagery. This report describes a procedure to implement the DLT equations and gives the Fortran code of computer programs for the DLT calibration of multicamera system and 3-D reconstruction of a single point from several images.</p>													
20. DISTRIBUTION/AVAILABILITY OF ABSTRACT <input checked="" type="checkbox"/> UNCLASSIFIED/UNLIMITED <input type="checkbox"/> SAME AS RPT. <input type="checkbox"/> DTIC USERS			21. ABSTRACT SECURITY CLASSIFICATION Unclassified										
22a. NAME OF RESPONSIBLE INDIVIDUAL Chief, Science Support Center			22b. TELEPHONE (Include Area Code) 205-255-6907		22c. OFFICE SYMBOL SGRD-UAX-SI								

Table of contents

	Page
List of figures	1
Introduction	3
Objectives	4
Methods	4
Results	6
Discussion	7
Conclusions	8
References	9
Appendix A. Listing of a subroutine to solve system of linear equations	12
Appendix B. Listing of subroutine to calibrate a camera	17
Appendix C. Listing of subroutine to reconstruct a point in 3D space	19

List of figures

Figure	Page
1. Flowchart of camera calibration procedure	10
2. Flowchart of 3D reconstruction of a single point.	11

Accession For	
NTIS CRA&I	<input checked="" type="checkbox"/>
DTIC TAB	<input type="checkbox"/>
Unannounced	<input type="checkbox"/>
Justification	
By	
Distribution /	
Availability Codes	
Dist	Avail and/or Special
A-1	

This page left blank intentionally.

Introduction

Photographic coverage is the simplest method of documenting the position and motion of objects in a laboratory environment. In many cases, it is the last resort for measuring dimensions, positions, and motion during an event when other measurement methods fail to provide needed data. Photogrammetry is the use of photographic images for the measurement of lengths and coordinates of an object. For many years, three-dimensional (3-D) analytical photogrammetry has been an essential tool in surveying and map making used for accurate measurements of land features. The technique required the use of expensive metric cameras to record aerial photographs and bulky comparators to reconstruct the 3-D coordinates of points on a land surface. The analytical methods used for the reconstruction were lengthy, complex, and iterative in nature, requiring the user to provide initial guesses of the coordinates being measured. Although highly accurate, these traditional methods are not readily applicable to laboratory measurements where stereo-base photography is commonly used for recording 3-D shapes.

In the early 1970s, researchers (Abdel-Aziz and Karara, 1971) simplified the complicated analytical procedure required for the reconstruction of 3-D coordinates from planar two-dimensional (2-D) images, and introduced the direct linear transformation (DLT) between the image and object spaces. The DLT reduced the complicated calibration procedures which were required to obtain inner and outer camera parameters, and eliminated the need for expensive metric cameras, thereby opening the door for relatively inexpensive nonmetric cameras to be used in 3-D photogrammetry. Further, the DLT allowed the use of convergent stereogrammetry which is suited ideally for laboratory close-range applications.

The simplicity of the DLT method in close-range photogrammetry spawned several technologies in the fields of biomechanics, sports medicine, orthopedics, automotive safety, robotics and other fields where accurate measurements of the 3-D coordinates and motion of an object were required. In one application where the mechanical properties of aortic membrane tissue were being investigated (Melvin, Mohan, Wineman, 1975), the DLT method was used to accurately measure the coordinates on the surface of aortic membrane as it inflated (Alem, Melvin, Holstein, 1978). In another application (Schneider et al., 1985), the coordinates of landmarks on the bodies of volunteer car drivers were measured in order to describe the size and shape of an "average" driver and to develop a crash manikin which is representative of the 50th percentile male driving population.

In a recent study, researchers at the U.S. Army Aeromedical Research Laboratory (USAARL) investigated the effects of rear-surface signature of 50-caliber body armor as it defeated a round. At the same time, the viscoelastic injury criterion was being advanced (Viano and Lau, 1988) and had gained wide acceptance. Briefly, the criterion predicts an injury risk from the product $V \cdot C$ of two measured parameters: the rate or velocity (V) of deformation of the chest, and the amount of compression (C) of the chest in the direction of impact. Clearly, in order to apply the viscoelastic injury criterion or evaluate the effects of the rear-surface on injury production, it would be necessary to measure the time-history of the rear surface as it deforms. This measurement then may be used to extract the speed of compression (V) of the chest wall, and the amount of

compression (C) of the chest. Because of the configuration of impact and the speed of deformation, noncontact 3-D high speed photogrammetry is the only practical method for accurate determination of the V*C product.

This report describes the method required to implement the DLT procedure for obtaining accurate static measurement of three-dimensional coordinates and provides a computer program which may be used for the calibration cameras pointed to a 3-D object space, and for the reconstruction of 3-D coordinates of point targets once the field has been calibrated. The method may be extended to many static and dynamic applications, including the measurement of the motion of rear-surface of the .50-caliber body armor as it defeats the penetration of a round.

Objectives

The objectives of this study are: (1) describe the direct linear transformation for close range photogrammetry; (2) present a procedure for the calibration of a 3-D object space using the DLT equations, and provide a computer program to implement the calibration procedure; and (3) present a procedure for the reconstruction of coordinates of a point target in the calibrated 3-D space using the DLT equations, and provide a computer program which implements the reconstruction procedure.

Methods

The direct linear transformation treats the entire chain of imaging components as a single system that reduces the 3-D object space to a 2-D image plane. Thus, we make no distinction between the camera proper and the projector. Instead, the entire camera-projector system is calibrated as a single unit. Further, because of the use of a single projector to produce images captured with different cameras, we refer to an imaging system as a "camera," with the understanding that this "camera" includes the projector as well.

For any such imaging system or "camera," it may be shown that the object space coordinates (x, y, z) of a point target are transformed to the image plane coordinates (u, v) according to the two linear equations:

$$\begin{aligned} u + xC_1 + yC_2 + zC_3 + C_4 + ux C_9 + uy C_{10} + uz C_{11} &= 0 \\ v + xC_5 + yC_6 + zC_7 + C_8 + vx C_9 + vy C_{10} + vz C_{11} &= 0 \end{aligned} \quad (1)$$

where $(C_1, C_2, \dots, C_{11})$ are constant coefficients which depend on the imaging system being used to convert the 3-D object (x, y, z) to a 2-D image (u, v) coordinates. The imaging system is said to be calibrated when its 11 DLT coefficients are determined.

In order to calibrate a camera, i.e., compute its 11 DLT coefficients, a sufficient number of control points are placed in the field-of-view of the camera to produce at least 11 equations in the 11 unknowns. A control point is a target whose (x, y, z) coordinates are known precisely in the 3-D object space. When its image is captured and projected by a camera system, the (u, v) image coordinates of a control point also become known. Therefore, the only unknowns in the 2 linear equations are 11 DLT coefficients. Since 2 equations may be written for each control point, a minimum of 6 control points are required to produce the system of linear equations in 11 unknowns.

The only constraint on the placement of the six control points is that they must not be coplanar to ensure the existence of a solution to the system of equations. With 6 control targets, 12 equations in 11 unknowns will be written. Although one equation may be dropped before solving for the unknowns, a least square solution is preferred to minimize the error. This implies that more control targets should be used to produce an overdetermined system of equations in the 11 unknowns which may then be solved by least-squares methods. Such practice reduces potential errors in determination of (x, y, z) and the reading of (u, v) coordinates, and produces more accurate results.

Given n control point targets ($n \geq 6$), we may write $2n$ equations in 11 unknowns. The resulting system of equations is written in matrix form as:

$$\begin{bmatrix} x_1 & y_1 & z_1 & 1 & 0 & 0 & 0 & 0 & u_1 x_1 & u_1 y_1 & u_1 z_1 \\ 0 & 0 & 0 & 0 & x_1 & y_1 & z_1 & 1 & u_1 x_1 & u_1 y_1 & u_1 z_1 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ x_i & y_i & z_i & 1 & 0 & 0 & 0 & 0 & u_i x_i & u_i y_i & u_i z_i \\ 0 & 0 & 0 & 0 & x_i & y_i & z_i & 1 & u_i x_i & u_i y_i & u_i z_i \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ x_n & y_n & z_n & 1 & 0 & 0 & 0 & 0 & u_n x_n & u_n y_n & u_n z_n \\ 0 & 0 & 0 & 0 & x_n & y_n & z_n & 1 & u_n x_n & u_n y_n & u_n z_n \end{bmatrix} \begin{Bmatrix} C_1 \\ C_2 \\ C_3 \\ \dots \\ \dots \\ C_{11} \end{Bmatrix} = \begin{Bmatrix} -u_1 \\ -v_1 \\ \dots \\ \dots \\ -u_i \\ -v_i \\ \dots \\ \dots \\ -u_n \\ -v_n \end{Bmatrix} \quad (2)$$

Equation (2) is an overdetermined system of linear algebraic equations in 11 unknowns which may be solved numerically with least squares methods. Figure 1 provides a flowchart of the procedure which may be followed for the calibration of two cameras.

Three-dimensional reconstruction simply means determination of the (x, y, z) position of a point in the calibrated 3-D object space, given the calibration constants of each camera, and the (u, v) image coordinates in each camera. This is the ultimate product of 3-D close range photogrammetry, that is, measures the coordinates of a point target in the laboratory 3-D space. To accomplish this, two or more cameras are used to capture the image of the point of interest at an

instant of time. Once the cameras have been calibrated as described earlier, the image of the same point target is digitized for each camera. This generates as many pairs of (u_k, v_k) coordinates of the same target as there are cameras. Thus, for the k-th camera, equations (1) are written as:

$$\begin{aligned} u_k + xC_{1,k} + yC_{2,k} + zC_{3,k} + C_{4,k} + u_kxC_{9,k} + u_kyC_{10,k} + u_kzC_{11,k} &= 0 \\ v_k + xC_{5,k} + yC_{6,k} + zC_{7,k} + C_{8,k} + v_kxC_{9,k} + v_kyC_{10,k} + v_kzC_{11,k} &= 0 \end{aligned} \quad (3)$$

where all terms are known except the three coordinates (x, y, z) of the target point. Therefore, a minimum of two cameras are required to produce sufficient number of equations (exactly four) which may be solved for the three unknowns. For practical reasons, several cameras are employed to cover the same field-of-view. Assuming that n cameras ($n \geq 2$) are available to observe the same point, equations (3) may be written for each camera then combined into the following matrix form:

$$\begin{bmatrix} (C_{1,1} + u_1C_{9,1}) & (C_{2,1} + u_1C_{10,1}) & (C_{3,1} + u_1C_{11,1}) \\ (C_{5,1} + v_1C_{9,1}) & (C_{6,1} + v_1C_{10,1}) & (C_{7,1} + v_1C_{11,1}) \\ \dots & \dots & \dots \\ \dots & \dots & \dots \\ (C_{1,k} + u_kC_{9,k}) & (C_{2,k} + u_kC_{10,k}) & (C_{3,k} + u_kC_{11,k}) \\ (C_{5,k} + v_kC_{9,k}) & (C_{6,k} + v_kC_{10,k}) & (C_{7,k} + v_kC_{11,k}) \\ \dots & \dots & \dots \\ \dots & \dots & \dots \\ (C_{1,m} + u_mC_{9,m}) & (C_{2,m} + u_mC_{10,m}) & (C_{3,m} + u_mC_{11,m}) \\ (C_{5,m} + v_mC_{9,m}) & (C_{6,m} + v_mC_{10,m}) & (C_{7,m} + v_mC_{11,m}) \end{bmatrix} \begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = \begin{Bmatrix} -(C_{4,1} + u_1) \\ -(C_{8,1} + v_1) \\ \dots \\ \dots \\ -(C_{4,k} + u_k) \\ -(C_{8,k} + v_k) \\ \dots \\ \dots \\ -(C_{4,m} + u_m) \\ -(C_{8,m} + v_m) \end{Bmatrix} \quad (4)$$

Equation (4) is an overdetermined system of $2n$ linear equations with (x, y, z) as the unknowns, to be solved using least-squares methods. Figure 2 provides a flowchart of the procedure which may be followed for the reconstruction of 3-D coordinates from the images of two cameras.

Results

Both calibration and reconstruction phases involve the solution of overdetermined system of linear equations using least-squares methods. The subroutine LLSQ listed in Appendix A is an implementation of such a method. It is coded for the Microsoft Fortran 5.0 compiler, but easily may be converted to other programming languages. The stability of the procedure depends on the selection of the control points and its accuracy on the number of points and the precision with which

their (x, y, z) and (u, v) coordinates were determined. A necessary constraint for the stability of the solution is for the control points to be nonplanar.

The calibration procedure outlined in Figure 1 is implemented in the subroutine DLTCAL. Appendix B provides a listing of the DLTCAL subroutine which sets up the equations and calls the LLSQ procedure to compute the 11 DLT coefficients. The subroutine is coded in free-form for the Microsoft Fortran 5.0 compiler, but easily may be modified for other languages. Input to the DLTCAL routine are the number of control points (at least six), the laboratory (x, y, z) coordinates of the control targets, and the (u, v) image coordinates of each target obtained by digitizing the image from the camera being calibrated. The subroutine returns the 11 coefficients for that camera.

The DLTCAL routine must be called for each camera being calibrated. Since all cameras must calibrate the same 3-D object space, only one array of control (x, y, z) coordinates must be used as input to the calibration routine, whereas the (u, v) array will vary from camera to camera, depending on the visibility of the target from that camera perspective.

Appendix C provides a listing of subroutine DLTREC which implements the 3-D reconstruction procedure outlined in Figure 2. The DLTREC subroutine sets up the equations and calls the LLSQ procedure to compute the three unknown coordinates. The routine is coded in free-form for the Microsoft Fortran 5.0 compiler, but easily may be modified for other languages. The reconstruction process, i.e., calls to the DLTREC routine, must be repeated for each target point. Input to DLTREC are the number of cameras (at least 2), the 11 DLT coefficients for each camera, and the image (u, v) coordinates of the target seen from each camera. The routine returns the reconstructed laboratory (x, y, z) coordinates of the target.

Discussion

The DLT method described here is a simple and accurate method for measurement of 3-D positions. It may be applied to a single point, to several points of a surface, or across several instants of time to perform 3-D motion analysis.

In practice, several cameras are calibrated simultaneously. Each camera produces its own set of DLT coefficients which are valid only for the physical setup of the imaging system that produced the (u, v) coordinates employed in that calibration. Once the imaging system which defines a camera has been disturbed because of repositioning of the camera mount, refocusing of the camera or projector, adjustment of projection angle or scale, or change of reference system for the image plane, the 11 DLT coefficients become invalid and must be determined anew by following the calibration process. To avoid repeated calibrations, cameras usually are locked into position for the duration of a study, and are removed only when the study is completed. Alternately, a fixed-based stereo system, with two or more cameras attached to a rigid but moveable structure, may be calibrated only once and will produce 3-D coordinates relative to the attachment structure.

To improve the precision of calibration and subsequent 3-D reconstruction, and to reduce the effects of lens distortion, the (u, v) coordinates should be measured in an image reference frame whose origin is at or near the optical center of the image. Since many digitizing tablets report coordinates relative to the lower left corner of the digitizing surface, it is necessary to transform the raw digitized data using a simple origin translation. This implies the need for a repeatable procedure to ensure precise redetermination of the same image center which was used in the calibration. In the absence of fiducial marks available in metric cameras, the procedure must rely on hardware features, such as the edges or corners of the imaging gate, which are always produced during the imaging process.

Researchers (Abdel-Aziz and Karara, 1974) have proposed various models to correct imaging distortions and to account for second-order terms neglected in the linearization of the object-image transformation. However, experience has shown that additional accuracy gained by the introduction of second-order corrections is insignificant when compared to potential improvement resulting from precision and care taken in the measurements of image coordinates, and does not justify the increase in equations and solution complexities.

Conclusions

A simple and accurate method has been provided to measure the 3-D coordinates of a point target in the laboratory using photographic coverage and the direct linear transformation. The full procedure leading to 3-D motion analysis will be presented in a separate report.

References

- Abdel-Aziz, Y. I., and Karara, H. M. 1971 "Direct linear transformation from comparator coordinates into object-space coordinates in close range photogrammetry." Urbana, Illinois: Proceedings of ASP Symposium on Close-Range Photogrammetry.
- Abdel-Aziz, Y. I., and Karara, H. M. 1974. Photogrammetric potentials of non-metric cameras. Urbana, IL: University of Illinois. Sponsored by National Science Foundation. Report No. UILU-ENG-74-2002.
- Alem, N. M., Melvin, J. W., and Holstein, G. L. 1978. Biomechanics applications of direct linear transformation in close-range photogrammetry. Proceedings of the sixth New England bioengineering conference, March 23-24 1978. New York: Pergamon Press.
- Melvin, J. W., Mohan, D., and Wineman, A. S. 1975. Mechanical characteristics of soft tissues at high strain rates. Proceedings of the 1975 biomechanics symposium. Troy, NY: American Society of Mechanical Engineers, AMD - Volume 10.
- Schneider, L. W., Robbins, D. H., Pflug, M. A., and Snyder, R. G. 1985. Design requirements and specifications of anthropometrically based design for an advanced adult anthropomorphic dummy family, Volume 1. Final report. Washington, DC: U.S. Department of Transportation, National Highway Traffic Safety Administration. DOT-HS-806-715.
- Viano, D. C., and Lau, I. V. 1988. A viscous tolerance criterion for soft tissue injury assessment. Journal of biomechanics. Vol. 21, No. 5, pp. 237-399.

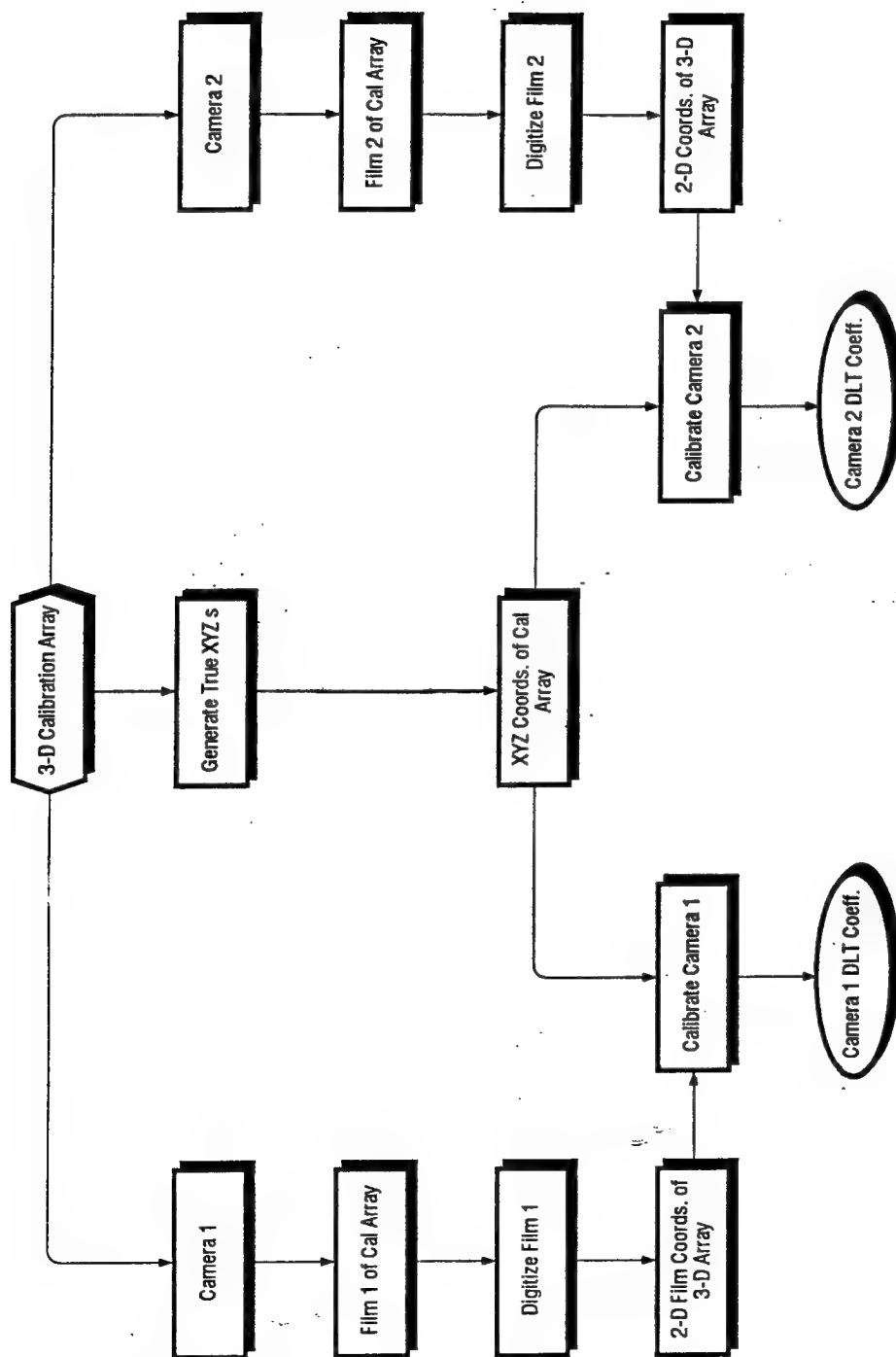


Figure 1. Flowchart of camera calibration procedure.

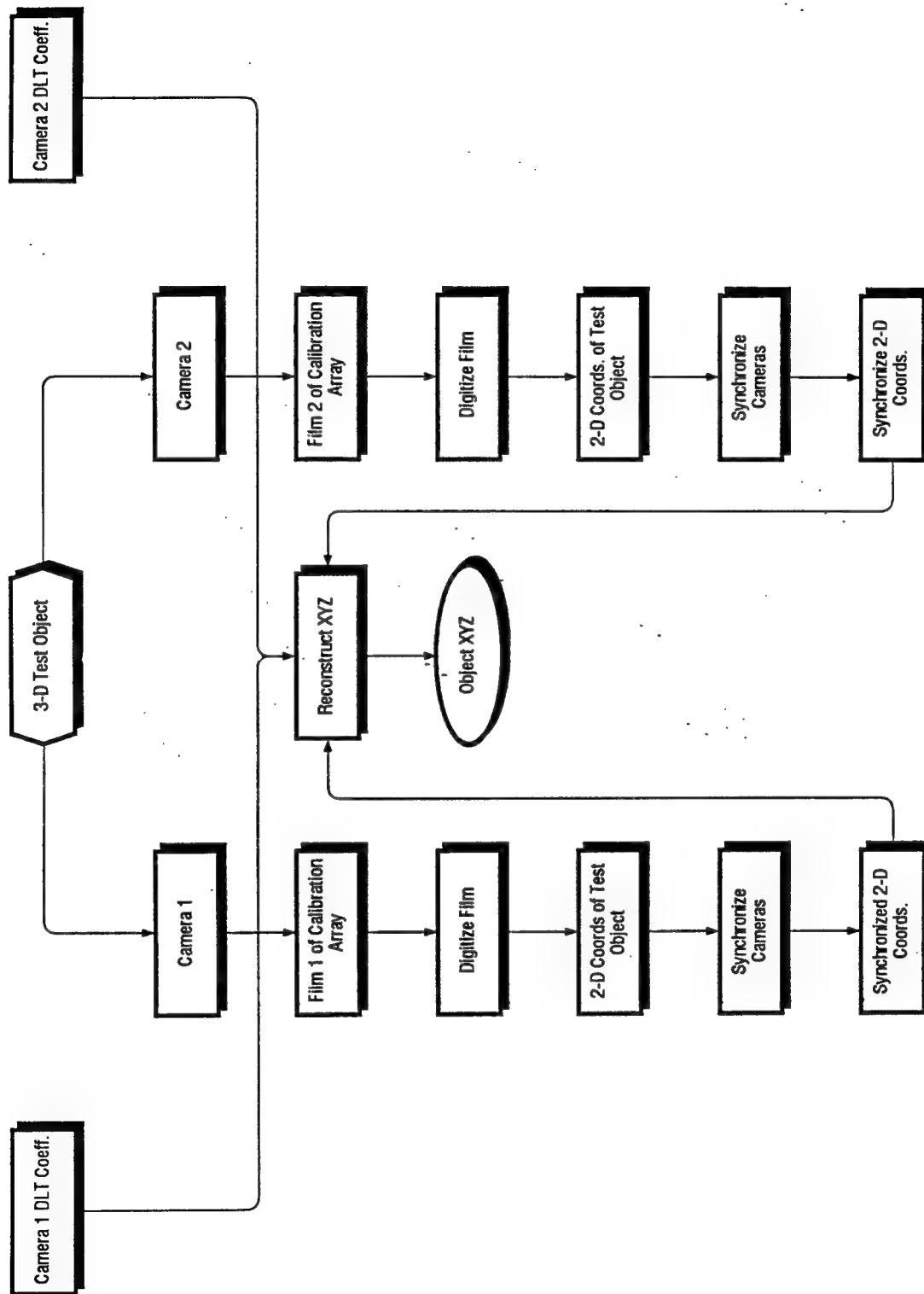


Figure 2. Flowchart of 3-D coordinates of a single point.

Appendix A.

Listing of a subroutine to solve system of linear equations.

```
*****
"  Coded for MS Fortran 5.0, free-form,    by N. Alem, USAARL,    October 1994
"
"  Purpose:      To solve the system of linear equations:
"
"                  [A] * {X} = {B}
"
"                by minimizing the euclidian norm of {B}-[A]*{X}, where [A]
"                is an M by N matrix with M not less than N.
"
"  Parameters:
"
"      A      ...  M by N coefficient matrix
"
"      B      ...  M by L right hand side matrix
"
"      M      ...  number of rows in matrices [A] and {B}
"
"      N      ...  number of unknown, i.e., number of columns of [A],
"                  which is also the number of rows in matrix {X}
"
"      L      ...  number of columns in {X} and {B}, (usually = 1)
"
"      X      ...  N by L solution matrix (usually N by 1 vector)
"
"      IPIV ...  Integer vector supplied by the use to receive
"                  information on column interchanges in matrix [A]
"
"      EPS    ...  Input parameter which specifies relative tolerance
"                  for determination of rank of matrix [A]
"
"      IER    ...  Error parameter.  IER=0 means successful solution.
"
"      AUX    ...  Auxiliary storage array of dimension max(2*N,L)
"                  On return, first L locations of AUX contain the
"                  resulting least squares.
"
*****

SUBROUTINE      LLSQ (A, B, M, N, L, X, IPIV, EPS, IER, AUX)

DIMENSION      A(*), B(*), X(*), IPIV(*), AUX(*)

IF (M .LT. N) THEN                                ! UNDER-DETERMINED SYSTEM
  IERR = -2                                         ! ERROR CODE
  RETURN
END IF
```

```

PIV = 0.                                ! INITIAL S(K) STORE IN AUX(K)
IEND = 0

DO K = 1, N
  IPIV (K) = K
  H = 0.
  IST = IEND + 1
  IEND = IEND + M

  DO I = IST, IEND
    H = H + A (I) * A (I)
  END DO

  AUX (K) = H
  IF (H .GT. PIV) THEN
    PIV = H
    KPIV = K
  END IF

END DO

IF (PIV .LE. 0.) THEN                    ! ZERO-MATRIX [A]
  IER = -1                               ! ERROR CODE
  RETURN
ENDIF

SIG = SQRT (PIV)                        ! TOLERANCE FOR CHECKING RANK OF [A]
TOL = SIG * ABS (EPS)

"***** DECOMPOSITION LOOP *****"
LM = L * M
IST = -M

DO K = 1, N
  IST = IST + M + 1
  IEND = IST + M - K
  I = KPIV - K

  IF (KPIV .GT. K) THEN                  ! EXCHANGE COLUMNS K AND KPIV
    H = AUX (K)
    AUX (K) = AUX (KPIV)
    AUX (KPIV) = H
    ID = I * M
    DO I = IST, IEND
      J = I + ID
      H = A (I)
      A (I) = A (J)
      A (J) = H
    END DO
  END IF

```



```

IF (K .GT. 1) THEN                                ! COMPUTATION OF PARAMETER SIG
  SIG = 0.
  DO I = IST, IEND
    SIG = SIG + A (I) * A (I)
  END DO
  SIG = SQRT (SIG)
  IF (SIG .LE. TOL) THEN                          ! TEST FOR SINGULARITY
    IER = K - 1                                  ! ERROR (K-1): RANK [A] LESS THAN N
    RETURN
  END IF
END IF

H = A (IST)
IF (H .LT. 0) SIG = -SIG

IPIV (KPIV) = IPIV (K)                            ! SAVE INTERCHANGE INFORMATION
IPIV (K) = KPIV
BETA = H + SIG                                     ! COMPUTE BETA AND VECTOR UK
A (IST) = BETA
BETA = 1. / (SIG * BETA)
J = N + K
AUX (J) = -SIG

IF (K .LT. N) THEN                                ! TRANSFORMATION OF MATRIX [A]
  PIV = 0.
  ID = 0
  JST = K + 1
  KPIV = JST

  DO J = JST, N
    ID = ID + M
    H = 0.
    DO I = IST, IEND
      II = I + ID
      H = H + A (I) * A (II)
    END DO
    H = BETA * H
    DO I = IST, IEND
      II = I + ID
      A(II) = A(II) - A(I) * H
    END DO
    II = IST + ID
    H = AUX(J) - A(II) * A(II)
    AUX(J) = H
    IF (H .GT. PIV) THEN
      PIV = H
      KPIV = J
    END IF
  END DO
ENDIF

```

```

DO J = K, LM, M                                ! TRANSFORM RIGHT HAND SIDE MATRIX B
  H = 0.
  IEND = J + M - K
  II = IST
  DO I = J, IEND
    H = H + A (II) * B (I)
    II = II + 1
  ENDDO
  H = BETA * H
  II = IST
  DO I = J, IEND
    B (I) = B (I) - A (II) * H
    II = II + 1
  END DO
END DO
END DO                                           ! END OF DECOMPOSITION LOOP

I = N                                           ! BACK SUBSTITUTION AND INTERCHANGE
LN = L * N
PIV = 1. / AUX (2 * N)
DO K = N, LN, N
  X (K) = PIV * B (I)
  I = I + M
END DO
IF (N .GT. 1) THEN
  JST = (N - 1) * M + N
  DO J = 2, N
    JST = JST - M - 1
    K = N + N + 1 - J
    PIV = 1. / AUX (K)
    KST = K - N
    ID = IPIV (KST) - KST
    IST = 2 - J
    DO K = 1, L
      H = B (KST)
      IST = IST + N
      IEND = IST + J - 2
      II = JST
      DO I = IST, IEND
        II = II + M
        H = H - A (II) * X (I)
      ENDDO
      I = IST - 1
      II = I + ID
      X (I) = X (II)
      X (II) = PIV * H
      KST = KST + M
    END DO
  END DO
ENDIF

```

```
IST = N + 1  
IEND = 0
```

! COMPUTATION OF LEAST SQUARES

```
DO J = 1, L  
  IEND = IEND + M  
  H = 0.
```

```
  IF (M .GT. N) THEN  
    DO I = IST, IEND  
      H = H + B(I) * B(I)  
    ENDDO  
    IST = IST + M  
  ENDIF
```

```
  AUX(J) = H  
ENDDO
```

```
IER = 0
```

```
RETURN  
END
```

Appendix B.

Listing of subroutine to calibrate a camera.

```
*****
"   Coded for MS Fortran 5.0, free-form,   by N. Alem, USAARL,       October 1994
"
"   Purpose:      Compute the 11 Direct Linear Transformation constants COF
"                  for a camera system. The COF are coefficient of the linear
"                  transformation which converts the three corrdinates (X,Y,Z)
"                  of a point to its image coordinates (U,V) in that camera.
"
"   Input:
"
"                  NPT      ... number of control points (minimum 6)
"
"                  XYZ(3,*) ... array containing the 3 laboratory coordinates
"                               of the NPT points, known with precision.
"
"                  UV(2,*)  ... array containing the 2 image coordinates of the
"                               NPT control points, digitized and referred to
"                               the optical center of the image.
"
"   Output:      COF(11)  ... eleven DLT coefficients for the camera.
"
"   Requires:    Subroutine LLSQ to solve the overdetermined system of linear
"                  equations with least squares method.
"
"   Note 1.      An error flag is returned via the * parameter to indicate
"                  that the linear least squares procedure (LLSQ) failed to
"                  converge to a solution. In this case, the parameter EPS
"                  which is required inside the LLSQ but defined here should be
"                  increased.
"
"   Note 2.      An error indication also occurs if the number of points NPT
"                  is less than 6 or greater than 50. If more than 50 points
"                  are used in the calibration, the dimension of AA() array
"                  should be increased to (22 * NPT), and BB() to (2 * NPT)
"
"   Note 3.      This calibration routine must be called for each camera,
"                  and must be repeated if any of the cameras used in the
"                  system has been altered or moved.
"
*****

SUBROUTINE      DLTCAL (NPT, XYZ, UV, COF, *)

REAL*4          XYZ(3,*), UV(2,*), COF(*)

DIMENSION       AA(1100), BB(100), AUX(22), IPIV(11)
REAL*4          EPS /1.E-15/
```

```

NEQ = 2 * NPT
I2 = 0

DO N = 1, NPT
  I1 = N
  I2 = I1 + 1

  AA (I1 + 0 * NEQ) = XYZ (1, N)
  AA (I1 + 1 * NEQ) = XYZ (2, N)
  AA (I1 + 2 * NEQ) = XYZ (3, N)
  AA (I1 + 3 * NEQ) = 1.0
  AA (I1 + 4 * NEQ) = 0.0
  AA (I1 + 5 * NEQ) = 0.0
  AA (I1 + 6 * NEQ) = 0.0
  AA (I1 + 7 * NEQ) = 0.0
  AA (I1 + 8 * NEQ) = UV (1, N) * XYZ (1, N)
  AA (I1 + 9 * NEQ) = UV (1, N) * XYZ (2, N)
  AA (I1 + 10 * NEQ) = UV (1, N) * XYZ (3, N)
  BB (I1) = - UV (1, N)

  AA (I2 + 0 * NEQ) = 0.0
  AA (I2 + 1 * NEQ) = 0.0
  AA (I2 + 2 * NEQ) = 0.0
  AA (I2 + 3 * NEQ) = 0.0
  AA (I2 + 4 * NEQ) = XYZ (1, N)
  AA (I2 + 5 * NEQ) = XYZ (2, N)
  AA (I2 + 6 * NEQ) = XYZ (3, N)
  AA (I2 + 7 * NEQ) = 1.0
  AA (I2 + 8 * NEQ) = UV (2, N) * XYZ (1, N)
  AA (I2 + 9 * NEQ) = UV (2, N) * XYZ (2, N)
  AA (I2 + 10 * NEQ) = UV (2, N) * XYZ (3, N)
  BB (I2) = - UV (2, N)

END DO

CALL LLSQ (AA, BB, NEQ, 11, 1, COF, IPIV, EPS, IER, AUX)

IF (IER .NE. 0) RETURN 1
RETURN
END

```

Appendix C.

Listing of subroutine to reconstruct a point in 3D space.

```
*****
"   Coded for MS Fortran 5.0, free-form,      by N. Alem, USAARL,  October 1994
"
"   Purpose:   Compute the 3 laboratory coordinates (x,y,z) of a target point
"              from its image coordinates (u,v) pairs obtained from two or
"              more cameras, given 11 DLT coefficients of each of the cameras.
"
"   Input:
"              NCAM      ... number of cameras (minimum 2)
"
"              COF(11,*) ... eleven DLT coefficients for each of the cameras.
"
"              UV(2,*)   ... array containing the image coordinates (u,v)
"                          pairs of the same point being reconstructed,
"                          observed and digitized in each camera image,
"                          and referred to the optical center of the image.
"
"   Output:
"              XYZ(3)    ... 3 laboratory coordinates (x,y,z) of the point
"                          being reconstructed.
"
"   Requires:
"              Subroutine LLSQ to solve the overdetermined system of linear
"              equations with least squares method.  LLSQ routine is listed
"              in Appendix A of this report.
"
"   Note 1.    An error flag is returned via the * parameter to indicate
"              that the linear least squares procedure (LLSQ) failed to
"              converge to a solution.  In this case, the parameter EPS
"              which is required inside the LLSQ but defined here should be
"              increased.
"
"   Note 2.    An error indication also occurs if the number of cameras NCAM
"              is less than 2 or greater than 9.  If more than 9 cameras were
"              used to digitize the same point, then the dimension of AA()
"              should be increased to (6 * NCAM), and BB() to (2 * NCAM).
"
*****

SUBROUTINE      RECXYZ (NCAM, COF, UV, XYZ, *)

REAL*4         COF(*), UV(2,*), XYZ(3,*)

DIMENSION      AA(54), BB(18), AUX(6), IPIV(3)
REAL*4         EPS /1.E-15/
```

```

IF (NCAM .LT. 2) RETURN 1
IF (NCAM .GT. 9) RETURN 1

NEQ = 2 * NCAM
I2 = 0

DO K = 1, NCAM

  I1 = K
  I2 = I1 + 1

  AA (I1 + 0 * NEQ) = COF (1, K) + UV (1, K) * COF ( 9, K)
  AA (I1 + 1 * NEQ) = COF (2, K) + UV (1, K) * COF (10, K)
  AA (I1 + 2 * NEQ) = COF (3, K) + UV (1, K) * COF (11, K)

  AA (I2 + 0 * NEQ) = COF (5, K) + UV (2, K) * COF ( 9, K)
  AA (I2 + 1 * NEQ) = COF (6, K) + UV (2, K) * COF (10, K)
  AA (I2 + 2 * NEQ) = COF (7, K) + UV (2, K) * COF (11, K)

  BB (I1) = - ( COF (4, K) - UV (1, K) )
  BB (I2) = - ( COF (4, K) - UV (2, K) )

END DO

CALL LLSQ (AA, BB, NEQ, 11, 1, XYZ, IPIV, EPS, IER, AUX)

if (IER .NE. 0) RETURN 1

RETURN
END

```

Initial distribution

Commander, U.S. Army Natick Research,
Development and Engineering Center
ATTN: SATNC-MIL (Documents
Librarian)
Natick, MA 01760-5040

Chairman
National Transportation Safety Board
800 Independence Avenue, S.W.
Washington, DC 20594

Commander
10th Medical Laboratory
ATTN: Audiologist
APO New York 09180

Naval Air Development Center
Technical Information Division
Technical Support Detachment
Warminster, PA 18974

Commanding Officer, Naval Medical
Research and Development Command
National Naval Medical Center
Bethesda, MD 20814-5044

Deputy Director, Defense Research
and Engineering
ATTN: Military Assistant
for Medical and Life Sciences
Washington, DC 20301-3080

Commander, U.S. Army Research
Institute of Environmental Medicine
Natick, MA 01760

Library
Naval Submarine Medical Research Lab
Box 900, Naval Sub Base
Groton, CT 06349-5900

Executive Director, U.S. Army Human
Research and Engineering Directorate
ATTN: Technical Library
Aberdeen Proving Ground, MD 21005

Commander
Man-Machine Integration System
Code 602
Naval Air Development Center
Warminster, PA 18974

Commander
Naval Air Development Center
ATTN: Code 602-B
Warminster, PA 18974

Commanding Officer
Armstrong Laboratory
Wright-Patterson
Air Force Base, OH 45433-6573

Director
Army Audiology and Speech Center
Walter Reed Army Medical Center
Washington, DC 20307-5001

Commander/Director
U.S. Army Combat Surveillance
and Target Acquisition Lab
ATTN: SFAE-IEW-JS
Fort Monmouth, NJ 07703-5305

Director
Federal Aviation Administration
FAA Technical Center
Atlantic City, NJ 08405

Director
Walter Reed Army Institute of Research
Washington, DC 20307-5100

Commander, U.S. Army Test
and Evaluation Command
Directorate for Test and Evaluation
ATTN: AMSTE-TA-M (Human Factors
Group)
Aberdeen Proving Ground,
MD 21005-5055

Naval Air Systems Command
Technical Air Library 950D
Room 278, Jefferson Plaza II
Department of the Navy
Washington, DC 20361

Director
U.S. Army Ballistic
Research Laboratory
ATTN: DRXBR-OD-ST Tech Reports
Aberdeen Proving Ground, MD 21005

Commander
U.S. Army Medical Research
Institute of Chemical Defense
ATTN: SGRD-UV-AO
Aberdeen Proving Ground,
MD 21010-5425

Commander
USAMRMC
ATTN: SGRD-RMS
Fort Detrick, Frederick, MD 21702-5012

HQ DA (DASG-PSP-O)
5109 Leesburg Pike
Falls Church, VA 22041-3258

Harry Diamond Laboratories
ATTN: Technical Information Branch
2800 Powder Mill Road
Adelphi, MD 20783-1197

U.S. Army Materiel Systems
Analysis Agency
ATTN: AMXSY-PA (Reports Processing)
Aberdeen Proving Ground
MD 21005-5071

U.S. Army Ordnance Center
and School Library
Simpson Hall, Building 3071
Aberdeen Proving Ground, MD 21005

U.S. Army Environmental
Hygiene Agency
ATTN: HSHB-MO-A
Aberdeen Proving Ground, MD 21010

Technical Library Chemical Research
and Development Center
Aberdeen Proving Ground, MD
21010-5423

Commander
U.S. Army Medical Research
Institute of Infectious Disease
ATTN: SGRD-UIZ-C
Fort Detrick, Frederick, MD 21702

Director, Biological
Sciences Division
Office of Naval Research
600 North Quincy Street
Arlington, VA 22217

Commandant
U.S. Army Aviation
Logistics School ATTN: ATSQ-TDN
Fort Eustis, VA 23604

Headquarters (ATMD)
U.S. Army Training
and Doctrine Command
ATTN: ATBO-M
Fort Monroe, VA 23651

IAF Liaison Officer for Safety
USAF Safety Agency/SEFF
9750 Avenue G, SE
Kirtland Air Force Base
NM 87117-5671

Naval Aerospace Medical
Institute Library
Building 1953, Code 03L
Pensacola, FL 32508-5600

Command Surgeon
HQ USCENTCOM (CCSG)
U.S. Central Command
MacDill Air Force Base, FL 33608

Director
Directorate of Combat Developments
ATTN: ATZQ-CD
Building 515
Fort Rucker, AL 36362

U.S. Air Force Institute
of Technology (AFIT/LDEE)
Building 640, Area B
Wright-Patterson
Air Force Base, OH 45433

Henry L. Taylor
Director, Institute of Aviation
University of Illinois-Willard Airport
Savoy, IL 61874

Chief, National Guard Bureau
ATTN: NGB-ARS
Arlington Hall Station
111 South George Mason Drive
Arlington, VA 22204-1382

AAMRL/HEX
Wright-Patterson
Air Force Base, OH 45433

Commander
U.S. Army Aviation and Troop Command
ATTN: AMSAT-R-ES
4300 Goodfellow Bouvelard
St. Louis, MO 63120-1798

U.S. Army Aviation and Troop Command
Library and Information Center Branch
ATTN: AMSAV-DIL
4300 Goodfellow Boulevard
St. Louis, MO 63120

Federal Aviation Administration
Civil Aeromedical Institute
Library AAM-400A
P.O. Box 25082
Oklahoma City, OK 73125

Commander
U.S. Army Medical Department
and School
ATTN: Library
Fort Sam Houston, TX 78234

Commander
U.S. Army Institute of Surgical Research
ATTN: SGRD-USM
Fort Sam Houston, TX 78234-6200

Air University Library
(AUL/LSE)
Maxwell Air Force Base, AL 36112

Product Manager
Aviation Life Support Equipment
ATTN: SFAE-AV-LSE
4300 Goodfellow Boulevard
St. Louis, MO 63120-1798

Commander and Director
USAE Waterways Experiment Station
ATTN: CEWES-IM-MI-R,
CD Department
3909 Halls Ferry Road
Vicksburg, MS 39180-6199

Commanding Officer
Naval Biodynamics Laboratory
P.O. Box 24907
New Orleans, LA 70189-0407

Assistant Commandant
U.S. Army Field Artillery School
ATTN: Morris Swott Technical Library
Fort Sill, OK 73503-0312

Mr. Peter Seib
Human Engineering Crew Station
Box 266
Westland Helicopters Limited
Yeovil, Somerset BA20 2YB UK

U.S. Army Dugway Proving Ground
Technical Library, Building 5330
Dugway, UT 84022

U.S. Army Yuma Proving Ground
Technical Library
Yuma, AZ 85364

AFFTC Technical Library
6510 TW/TSTL
Edwards Air Force Base,
CA 93523-5000

Commander
Code 3431
Naval Weapons Center
China Lake, CA 93555

Aeromechanics Laboratory
U.S. Army Research and Technical Labs
Ames Research Center, M/S 215-1
Moffett Field, CA 94035

Sixth U.S. Army
ATTN: SMA
Presidio of San Francisco, CA 94129

Commander
U.S. Army Aeromedical Center
Fort Rucker, AL 36362

Strughold Aeromedical Library
Document Service Section
2511 Kennedy Circle
Brooks Air Force Base, TX 78235-5122

Dr. Diane Damos
Department of Human Factors
ISSM, USC
Los Angeles, CA 90089-0021

U.S. Army White Sands
Missile Range
ATTN: STEWS-IM-ST
White Sands Missile Range, NM 88002

Director, Airworthiness Qualification Test
Directorate (ATTC)
ATTN: STEAT-AQ-O-TR (Tech Lib)
75 North Flightline Road
Edwards Air Force Base, CA 93523-6100

Ms. Sandra G. Hart
Ames Research Center
MS 262-3
Moffett Field, CA 94035

Commander
USAMRMC
ATTN: SGRD-UMZ
Fort Detrick, Frederick, MD 21702-5009

Commander
U.S. Army Health Services Command
ATTN: HSOP-SO
Fort Sam Houston, TX 78234-6000

U. S. Army Research Institute
Aviation R&D Activity
ATTN: PERI-IR
Fort Rucker, AL 36362

Commander
U.S. Army Safety Center
Fort Rucker, AL 36362

U.S. Army Aircraft Development
Test Activity
ATTN: STEBG-MP-P
Cairns Army Air Field
Fort Rucker, AL 36362

Commander
USAMRMC
ATTN: SGRD-PLC (COL R. Gifford)
Fort Detrick, Frederick, MD 21702

TRADOC Aviation LO
Unit 21551, Box A-209-A
APO AE 09777

Netherlands Army Liaison Office
Building 602
Fort Rucker, AL 36362

British Army Liaison Office
Building 602
Fort Rucker, AL 36362

Italian Army Liaison Office
Building 602
Fort Rucker, AL 36362

Directorate of Training Development
Building 502
Fort Rucker, AL 36362

Chief
USAHEL/USAAVNC Field Office
P. O. Box 716
Fort Rucker, AL 36362-5349

Commander, U.S. Army Aviation Center
and Fort Rucker
ATTN: ATZQ-CG
Fort Rucker, AL 36362

Dr. Sehchang Hah
Dept. of Behavior Sciences and
Leadership, Building 601, Room 281
U. S. Military Academy
West Point, NY 10996-1784

Canadian Army Liaison Office
Building 602
Fort Rucker, AL 36362

German Army Liaison Office
Building 602
Fort Rucker, AL 36362

French Army Liaison Office
USAAVNC (Building 602)
Fort Rucker, AL 36362-5021

Australian Army Liaison Office
Building 602
Fort Rucker, AL 36362

Dr. Garrison Rapmund
6 Burning Tree Court
Bethesda, MD 20817

Commandant, Royal Air Force
Institute of Aviation Medicine
Farnborough, Hampshire GU14 6SZ UK

Defense Technical Information
Cameron Station, Building 5
Alexandra, VA 22304-6145

Commander, U.S. Army Foreign Science
and Technology Center
AIFRTA (Davis)
220 7th Street, NE
Charlottesville, VA 22901-5396

Commander
Applied Technology Laboratory
USARTL-ATCOM
ATTN: Library, Building 401
Fort Eustis, VA 23604

Commander, U.S. Air Force
Development Test Center
101 West D Avenue, Suite 117
Eglin Air Force Base, FL 32542-5495

Aviation Medicine Clinic
TMC #22, SAAF
Fort Bragg, NC 28305

Dr. H. Dix Christensen
Bio-Medical Science Building, Room 753
Post Office Box 26901
Oklahoma City, OK 73190

Commander, U.S. Army Missile
Command
Redstone Scientific Information Center
ATTN: AMSMI-RD-CS-R
/ILL Documents
Redstone Arsenal, AL 35898

Aerospace Medicine Team
HQ ACC/SGST3
162 Dodd Boulevard, Suite 100
Langley Air Force Base,
VA 23665-1995

U.S. Army Research and Technology
Laboratories (AVSCOM)
Propulsion Laboratory MS 302-2
NASA Lewis Research Center
Cleveland, OH 44135

Commander
USAMRMC
ATTN: SGRD-ZC (COL John F. Glenn)
Fort Detrick, Frederick, MD 21702-5012

Dr. Eugene S. Channing
166 Baughman's Lane
Frederick, MD 21702-4083

U.S. Army Medical Department
and School
USAMRDALC Liaison
ATTN: HSMC-FR
Fort Sam Houston, TX 78234

NVESD
AMSEL-RD-NV-ASID-PST
(Attn: Trang Bui)
10221 Burbeck Road
Fort Belvoir, VA 22060-5806

CA Av Med
HQ DAAC
Middle Wallop
Stockbridge, Hants S020 8DY UK

Dr. Christine Schlichting
Behavioral Sciences Department
Box 900, NAVUBASE NLON
Groton, CT 06349-5900

Commander
Aviation Applied Technology Directorate
ATTN: AMSAT-R-TV
Fort Eustis, VA 23604-5577

COL Yehezkel G. Caine, MD
Surgeon General, Israel Air Force
Aeromedical Center Library
P. O. Box 02166 I.D.F.
Israel

HQ ACC/DOHP
205 Dodd Boulevard, Suite 101
Langley Air Force Base,
VA 23665-2789

41st Rescue Squadron
41st RQS/SG
940 Range Road
Patrick Air Force Base,
FL 32925-5001

48th Rescue Squadron
48th RQS/SG
801 Dezonias Road
Holloman Air Force Base,
NM 88330-7715

HQ, AFOMA
ATTN: SGPA (Aerospace Medicine)
Bolling Air Force Base,
Washington, DC 20332-6128

ARNG Readiness Center
ATTN: NGB-AVN-OP
Arlington Hall Station
111 South George Mason Drive
Arlington, VA 22204-1382

35th Fighter Wing
35th FW/SG
PSC 1013
APO AE 09725-2055

66th Rescue Squadron
66th RQS/SG
4345 Tyndall Avenue
Nellis Air Force Base, NV 89191-6076

71st Rescue Squadron
71st RQS/SG
1139 Redstone Road
Patrick Air Force Base,
FL 32925-5000

Director
Aviation Research, Development
and Engineering Center
ATTN: AMSAT-R-Z
4300 Goodfellow Boulevard
St. Louis, MO 63120-1798

Commander
USAMRMC
ATTN: SGRD-ZB (COL C. Fred Tyner)
Fort Detrick, Frederick, MD 21702-5012

Commandant
U.S. Army Command and General Staff
College
ATTN: ATZL-SWS-L
Fort Leavenworth, KS 66027-6900

ARNG Readiness Center
ATTN: NGB-AVN-OP
Arlington Hall Station
111 South George Mason Drive
Arlington, VA 22204-1382

Director
Army Personnel Research Establishment
Farnborough, Hants GU14 6SZ UK

Dr. A. Kornfield
895 Head Street
San Francisco, CA 94132-2813

ARNG Readiness Center
ATTN: NGB-AVN-OP
Arlington Hall Station
111 South George Mason Drive
Arlington, VA 22204-1382

Cdr, PERSCOM
ATTN: TAPC-PLA
200 Stovall Street, Rm 3N25
Alexandria, VA 22332-0413

HQ, AFOMA
ATTN; SGPA (Aerospace Medicine)
Bolling Air Force Base,
Washington, DC 20332-6188